Plate Analysis of Corrucomb® – Steel Skin Sandwich Panel in Bending

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Abstract

Plate theory was employed to analyze the Corrucomb – steel skin sandwich structure in bending. Modulus of elasticity of a uniform plate was first derived by ignoring the contribution of the core, and then further reduced into an apparent modulus by incorporating the shear deflection of the core. This apparent modulus is used to calculate the plate deflection.

Bending stress of the skin as well as the reacting force and shear stress at the plate edges was also analyzed, for both simply supported and edges fixed boundary conditions. The reacting force translates into compressive stress to the core which has to be sustained by the Corrucomb, together with shear stress.

Finally, examples were used to show the calculation of deflection, bending stress, compressive stress and shear stress, as a function of steel skin thickness.

Introduction

Innomation Inc has developed a proprietary process of corrugating phenolic resin saturated kraft paper into a continuous ribbon of Corrucomb cells with depth over ½” and pitch over ¾”. The ribbon is then assembled into Corrucomb block and cut into dimensions for sandwich lay-up. The two facings for the sandwich panels are tentatively thin steel skins. The finished panel could be as thick as 2 feet.

The sandwich components can be used as second floor deck, roof and wall envelop panels in office and other commercial buildings. Furthermore, Innomation intends to join the edges of individual panels to convert such surface system from beam into plate construction, to increase the loading capacity.

This paper presents the analysis of the sandwich plate for such applications. Specifically, the bending deflection and the bending stress of the skin are approximated using the small deflection thin plate theory.

Analysis Assumptions

Because the steel skin is assumed to carry the bending load, the sandwich panel is assumed isotropic even though the Corrucomb may not be. To use the plate theory, the sandwich panel is first "translated" into an isotropic uniform material of the same dimension but with a reduced young's modulus (E_{true}) by the following equation:

$$E_{true} = E(1-c^3/d^3) \qquad (1)$$

where E is the modulus of elasticity of the steel, c is the core thickness and d is the thickness of the sandwich panel.

Even though the Corrucomb may not contribute in carrying the bending load, it will contribute to the overall deflection due to its relative low shear modulus, which may be significant. In order to use the plate theory, the modulus of Equation (1) is further reduced and new modulus (called apparent modulus $E_{apparent}$) is calculated by the following equation (APA 2004):

$$E_{apparent} = E_{true} /(1+384E_{true}I/(5K\ell^2)) \qquad (2)$$

for uniformly distributed loading, simply supported beam, in which K is the shear deflection coefficient, I is the moment of inertia, and ℓ is the span. It can be reduced to:

$$E_{apparent} = E_{true} /(1+0.96E_{true}d^2/(G\ell^2)) \qquad (3)$$

where G is the shear modulus of the core.

As an example, Table 1 lists the calculation results of true modulus, apparent modulus, and the ratio of the two, as a function of steel skin thickness for d=24", ℓ=600", E=30x10^6 psi and G=1000 psi. MOE ratio is not constant, rather decreases as skin thickness increases.

Table 1 – Calculation results MOE and deflection at 100 lbs/ft^2

Skin Thickness (inches)	True MOE (psi)	Apparent MOE (psi)	MOE Ratio	Deflection (inches)	
				True MOE	Apparent MOE
0.0200	149750	121746	0.81	1.930	2.370
0.0400	299001	204898	0.68	0.964	1.410
0.0600	447753	265296	0.59	0.644	1.090
0.0800	596009	311155	0.52	0.484	0.927
0.1000	743767	347161	0.47	0.388	0.830
0.1200	891030	376180	0.42	0.324	0.766
0.1400	1037797	400067	0.39	0.278	0.721
0.1600	1184071	420072	0.35	0.243	0.686

Plate Analysis in Bending

Case 1: Uniform loaded and simply supported rectangular plate

a. Plate deflection

The maximum deflection (ω) occurs at the center of the plate and is given by the following Equation:

$$\omega = \alpha q a^4/D \qquad (4)$$

where D = $E_{apparent}d^3/\{12(1-\upsilon^2)\}$, the flexural rigidity of the plate, υ is the poison ratio, q is the uniform loading pressure, and α is a function of ratio b/a, and b and a are respectively the length of the long and short sides of the plate. Parameter α is provided in Table 8 of Timoshenko and Krieger's book (1959).

b. Bending stress

The maximum bending moment (M_{max}) also occurs at the middle of the plate and is given by:

$$M_{max} = \beta q a^2 \qquad (5)$$

where β is a function of b/a and also provided by Timoshenko and Krieger (Table 8). The maximum stress of the skin of the sandwich panel then can be approximated by

$$\sigma = \beta q a^2/td \qquad (6)$$

where t is the skin thickness.

c. *Compression Stress*

For simply supported plate, the reactive load at the supports is not uniformly distributed. The maximum load occurs at the center of each side of the plate, and the maximum reaction (R_{max}) per unit length is given by:

$$R_{max} = \delta q a \qquad (7)$$

in which δ is dependent of the ratio b/a and given by Timoshenko and Krieger (Table 8). This reaction force translates into compressive stress to the core and the bearing support has to be designed accordingly.

d. *Shear Stress*

The shear force (Q) per unit length is also maximum at the center of the four edges of the plate. The magnitude is calculated by the following equation:

$$Q = \gamma q a \quad (8)$$

and the shear stress (τ) is calculated by:

$$\tau = \gamma\, qa/d \quad (9)$$

where γ is a numerical factor dependent on the ratio of b/a and also given in Table 8 of Timoshenko and Krieger (1959). The Corrucomb has to be able to carry this magnitude of shear stress.

Case 2: Uniform loaded and edges fixed rectangular plate

e. Plate deflection and bending stress

Plate deflection is still given by Equation (4) but the coefficient α is given in Table 35 of the Timoshenko and Krieger's book (1959) and is much smaller because of the fixed edges.

The equation for the stress is still the same (Equation 6) as for the simply supported plate, but again the coefficient is provided in table 35. Furthermore, the maximum moment and therefore the maximum bending stress occurs at the center of the four edges.

f. Compressive and shear stress

Because the edges are fixed, the reacting force and shear stress are assumed uniformly distributed along the edges. Therefore, the following equations are valid:

$$R_{max} = qab/(2a+2b) \quad (10)$$

$$\tau = qab/(2d(a+b)) \quad (11)$$

Examples

Innomation is in the process of building a demonstration office using the steel faced Corrucomb sandwich panels. The office will be a 2-story building of 50'x60'x28', with roof being 50'x60', second floor deck 50'x50', and wall either 28'x50' or 28'x60'. The following calculation shows the plate deflection and stresses as a function of steel skin thickness, for both simply supported and edges fixed boundary conditions.

1. Second floor deck 50'x50'

The total depth d of the second floor deck is chosen to be 24". For simply supported plate with a uniform live load of 100 lbs/ft^2, Table 1 shows the plate deflection as a function of steel skin thickness, assuming E = 30 x10^6 psi and υ =0.3. Table 1 also shows the deflection using both E_{true} and $E_{apparent}$ as the modulus for the calculation. The ratio of deflection mirrors the MOE ratio, and deflection will be underestimated if the shear deflection is ignored.

Table 2 lists comparison of the deflections between simply supported and edges fixed boundary conditions at 100 lbs/ft^2 uniform loading. Deflection is greatly reduced by fixing the edges of the plate.

Table 2 – Deflection at 100 lbs/ft^2 uniform loading

Skin Thickness (inches)	Simply Supported (inches)	Edges Fixed (inches)
0.0200	2.370	0.738
0.0400	1.410	0.438
0.0600	1.090	0.339
0.0800	0.927	0.289
0.1000	0.830	0.259
0.1200	0.766	0.239
0.1400	0.721	0.225
0.1600	0.686	0.214

In the real world, the boundary condition will be somewhere in between the simply supported and edges fixed scenarios. Building code requires a deflection limit less than 1/360 of the span for the live load, which equals to 1.67" for 50' span. To meet this deflection limit, a skin thickness of 0.0400" is required.

In order to be 100 lbs/ft^2 certified, the floor needs to survive a 250 lbs/ft^2 loading (2.5 times the design load). Table 3 shows the bending stresses of the steel skin using Equation (6) at this loading level for both boundary conditions. Stresses increased slightly due to fixed edges, but if a yielding strength of 50 ksi is assumed, a skin thickness of 0.0400" is sufficient.

Table 3 – Estimated stress at 250 lbs/ft^2 uniform loading

Skin Thickness (inches)	Simply Supported (psi)	Edges Fixed (psi)
0.0200	62369	66796
0.0400	31184	33398
0.0600	20789	22265
0.0800	15592	16699
0.1000	12473	13359
0.1200	10394	11132
0.1400	8909	9542
0.1600	7796	8349

Table 4 shows the estimation of compressive force and shear stress induced to the Corrucomb at 250 lbs/ft^2 loading, for both boundary conditions. It is believed that the shear strength of the core exceeds the shear stress. If a compressive strength of 50 psi is assumed, the base of the angle steel needs to be >8.75".

Table 4 – Compressive force and shear stress at 250 lbs/ft^2 uniform loading

Load	Simply supported Plate	Edges Fixed Plate
Compressive Force (lbs/ft)	5250	3125
Shear Stress (psi)	14.67	10.85

2. Roof deck 50'x60'

The total depth d of the roof deck is chosen to be 18". Table 5 lists the deflections at 40 lbs/ft^2 uniform loading for both simply supported and edges fixed boundary conditions. To meet the deflection limit of 1/360 of the span, a skin thickness of 0.0400" is required, assuming simply supported edges.

Table 5 – Deflection at 40 lbs/ft^2 uniform loading

Skin Thickness (inches)	Simply Supported (inches)	Edges Fixed (inches)
0.0200	2.237	0.683
0.0400	1.292	0.394
0.0600	0.972	0.300
0.0800	0.806	0.246
0.1000	0.708	0.218
0.1200	0.653	0.205
0.1400	0.611	0.191
0.1600	0.570	0.177

In order to be 40 lbs/ft^2 certified, the roof needs to survive a 100 lbs/ft^2 loading. Table 6 shows the bending stresses of the steel skin using Equation (6) at this loading level for both boundary conditions. If a yielding strength of 50 ksi is assumed, a skin thickness of 0.0200" is sufficient.

Table 6 – Estimated stress at 100 lbs/ft^2 uniform loading

Skin Thickness (inches)	Simply Supported (psi)	Edges Fixed (psi)
0.0200	43541	44374
0.0400	21770	22187
0.0600	14513	14791
0.0800	10885	11093
0.1000	8708	8874
0.1200	7256	7395
0.1400	6220	6339
0.1600	5442	5546

Table 7 shows the estimation of compressive force and shear stress induced to the Corrucomb at 100 lbs/ft^2 loading, for both boundary conditions. It is believed that the shear strength of the core exceeds the shear stress. If a compressive strength of 50 psi is assumed, the base of the angle steel needs to be >3.8".

Table 7 – Compressive force and shear stress at 100 lbs/ft^2 uniform loading

Load	Simply Supported Plate	Edges Fixed Plate
Compressive Force (lbs/ft)	2275	1364
Shear Stress (psi)	8.79	6.31

3. Wall of 14'x60'

The wall thickness is chosen to be 5.5", corresponding to 2"x6" framing lumber. Because the second floor deck divides the wall height in half, the analysis is performed on only half of the wall, i.e., 60'x14' panel.

The design wind load is 15 lbs/ft^2 @ wind speed of 90 mph for Mid West of the United States. Table 8 shows the calculation of deflection for both simply supported and edges fixed boundary conditions. . A skin thickness of 0.0200" is more than sufficient to meet the deflection limit of 1/240" of the span.

Table 8 – Deflection at 15 lbs/ft^2 uniform loading

Skin Thickness (inches)	Simply Supported (inches)	Edges Fixed (inches)
0.0200	0.180	0.036
0.0400	0.126	0.026
0.0600	0.108	0.022
0.0800	0.099	0.02
0.1000	0.094	0.019
0.1200	0.091	0.018
0.1400	0.088	0.018
0.1600	0.086	0.017

Table 9 lists the bending stresses at 37.5 lbs/ft^2 (2.5 times of the design wind load) and 400 lbs/ft^2 tornado load. A skin thickness of 0.0200" is more than sufficient for the wind load, but 0.0400" is required for the Tornado load, if a yielding strength of 50 ksi is assumed.

Table 9 – Bending stresses at two loading levels and two boundary conditions

Skin Thickness (inches)	Stress at 37.5 psf load		Stress at Tornado load 400 psf	
	Simply Supported	Edges Fixed	Simply Supported	Edges Fixed
0.0200	8271	5588	88224	59615
0.0400	4135	2794	44112	29807
0.0600	2757	1862	29408	19871

0.0800	2067	1397	22056	14903
0.1000	1654	1117	17644	11923
0.1200	1378	931	14704	9935
0.1400	1181	798	12603	8516
0.1600	1033	698	11028	7451

Table 10 lists the estimation of compressive force and shear stress induced to the Corrucomb at 37.5 psf and 400 psf loadings, for both boundary conditions. It is believed that the shear stress induced by the Tornado force exceeded the shear strength of the core. In order for the system to survive the Tornado force, the Corrucomb needs to be reinforced somehow, which is beyond the scope of this paper.

If a compressive strength of 50 psi is assumed, the base of the support at the edges needs to be >4.6” for the Tornado force.

Table 10 – Compressive force and shear stress at 37.5 psf and 400 psf loading

Load	37.5 psf Loading		400 psf loading	
	Simply Supported	Edges Fixed	Simply Supported	Edges Fixed
Compressive Force (lbs/ft)	263	202	2800	2154
Shear Stress (psi)	4.0	3.1	42.3	32.6

Conclusions

The analysis showed that a skin thickness of 0.0400” is required for both second floor deck and roof panels for the demonstration office building, for the required design load. A skin thickness of 0.0200” is sufficient for the wall for the design wind load, but a thickness of 0.0400” is required for the tornado force. Reinforcement of shear capacity of the core is also required for the tornado design.

References

APA - The Engineered Wood Association. 2004. True (shear free) and apparent moduli of elasticity.

Timoshenko, S. P. and S. W. Krieger. 1959. Theory of plates and shells. Second Edition. McGraw-Hill Book Company.